Effect of Time Delay on the Stability of Reversing Semitrailer ⋆

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Abstract: In this study, the reverse motion of an autonomous truck–semitrailer combination along a circular path is investigated. To do this, the single-track kinematic model is applied. As the time delay in the control loop (assuming a simple proportional control law) is considered, the semi-discretization method is used for the stability analysis. The significant effect of the time delay on stability is emphasized with nonlinear simulations.

Keywords: truck–semitrailer, time delay, path-following, reversing, stability of delay systems

1. INTRODUCTION

Time is one of the most important factors for freight transport companies. Since the travel speed on the roads is limited, it may be a relevant task to reduce dwell time in the warehouse (Ardhi et al., 2019). It can be possible by developing autonomous features for truck–trailer systems in order to realize complicated parking maneuvers faster. In this paper, the path-following problem of a truck–semitrailer in reverse motion is presented, which is unavoidable for the implementation of complicated docking maneuvers. The vehicle system is modeled by the commonly used single-track kinematic model with the assumption of rigid wheels. The steering angle of the truck's front axle is controlled by a linear state feedback controller with the consideration of time delay in the control loop. Due to the localization problem of an autonomous truck (mostly based on image processing methods), the time delay may be significant. It will be shown that it may be essential to consider time delay in the model in order to stabilize and accelerate the docking process of articulated vehicles.

2. METHOD OF MODELING

The simplified mechanical model of the truck–semitrailer is shown in Fig. 1. Due to the single-track model, rigid wheels are located at points F, R and T, representing the front and rear axle of the truck, and the axle of the trailer, respectively. The truck and the trailer are connected at the kingpin K. The circular path-following task can be described with the lateral error e, the angle error θ and the relative yaw angle φ (based on a coordinate transformation detailed in Qin et al. (2022)). The equations of motion are derived using the kinematic constraints of the rolling rigid wheels; namely, the velocity vectors of points F, R and T are parallel to the proper wheel planes. Another kinematic constraint is that the longitudinal velocity V of the rear axle R is constant. In this study, the curvature κ of the path is also constant in order to accomplish the stability analysis, however, the model can handle varying curvature as well.

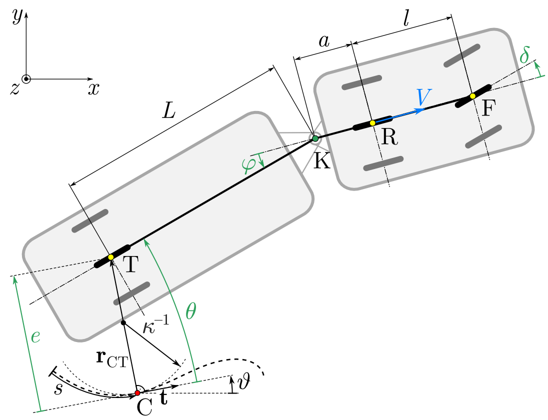

Fig. 1. Mechanical model of the truck–semitrailer.

The vehicle system is actuated via the steering angle δ of the front axle F. Furthermore, our model takes into account the dynamics of the steering mechanism by

$$\dot{\delta} = \omega\,, \quad \dot{\omega} = -p\,(\delta - \delta_{\rm des}) - d\omega\,, \tag{1}$$

where ω is the steering rate; p and d are the control gains of the steering servo. The desired steering angle is calculated by the control law:

$$\delta_{\rm des}(t) = \delta_{\rm ff} + \delta_{\rm fb}(t), \tag{2}$$

where $\delta_{\rm ff}$ is the feedforward term. The time-delayed feedback term is

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$$\delta_{\text{fb}}(t) = -P_e\, e(t-\tau) - P_\theta\, \theta(t-\tau) \\ - P_\varphi\big(\varphi(t-\tau) - \varphi^\star\big), \tag{3}$$

where τ refers to the time delay. The control gains relate to lateral error, angle error, and relative yaw angle error. Note that the feedforward term δ_{ff} and the desired relative yaw angle $\varphi^\star$ are the steady-state solutions, however, for constant curvature, they can be determined by geometric consideration. The numerical values of the parameters are detailed in Table 1.

Table 1. Parameter values

Parameter	Numerical value
a	-0.8 m
l	3.5 m
L	10 m
κ	0.1 m^{-1}
p	300 1/s^2
d	34.6 1/s

3. STABILITY AND SIMULATION RESULTS

The stability analysis of this time-delayed system is implemented using the semi-discretization method (Insperger and Stépán, 2011). The effect of time delay on stability is introduced on stability charts and further investigated by the results of nonlinear simulations.

Stability chart in the plane of the control gains (P_θ, P_φ) is shown in Fig. 2. A model that neglects the effect of time delay provides the gray stable area for reversing velocity $V = -1.5$ m/s with the most stable gain setup (according to the stability criterion based on the characteristic exponents) denoted by the black cross. Our model that takes a significant time-delay $\tau = 0.5$ s into account produces the stable domain marked by dark blue for velocity $V = -1.5$ m/s and light blue for $V = -1.25$ m/s.

Based on the remarkable differences in the stability chart, the effect of time delay on stability is further investigated by simulations, see Fig. 3. Take a scenario when the desired reversing velocity is $V = -1.5$ m/s, the path curvature is $\kappa = 0.1$ m^{-1} and the actual time delay is $\tau = 0.5$ s. Neglecting time delay in the model, the most stable control gain setup (black cross in Fig. 2) leads to the reverse motion denoted by yellow dashed line. Operating at a reduced velocity $V = -1.25$ m/s while using the same control gains leads to the motion marked by red thin line. Finally, applying the most stable control gain setup related to the model that considers the effect of time delay $\tau = 0.5$ s (white cross in Fig. 2) with velocity $V = -1.5$ m/s leads to the reverse motion represented by the blue thick line.

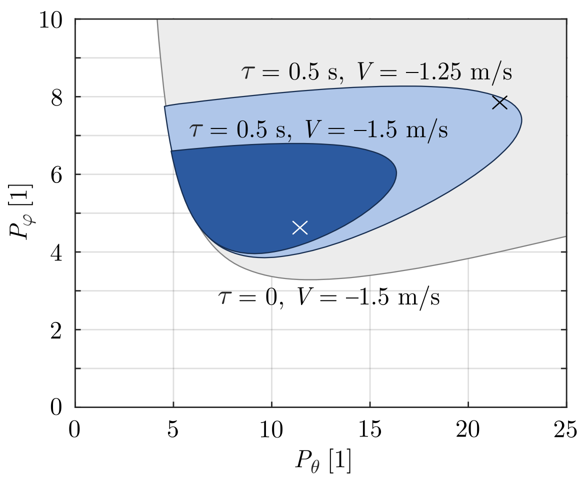

Fig. 2. Stability chart for three distinct scenarios. Control gain related to the lateral error is fixed at $P_e = -5$ rad/m and the path curvature is $\kappa = 0.1$ m^{-1}. The filled areas represent the control gain setups of stable reverse motion for different time delay and vehicle speed. Crosses refer to most stable gain setups.

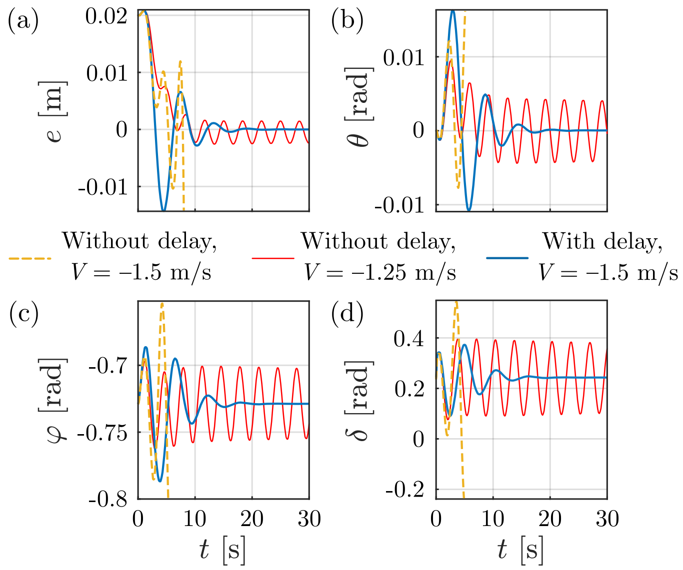

Fig. 3. Simulation results for three different cases. The graphs provide the time series of (a) lateral error e, (b) angle error θ, (c) relative yaw angle φ and (d) steering angle δ. The initial condition is set to the steady state solution except for the lateral error $e = 0.02$ m.

4. CONCLUSION

In conclusion, tuning the controller without considering the time delay may lead unstable motion. Although reducing velocity can help stabilize the reverse motion, better performance can be ensured when time delay is accounted.

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